



Special Relativity . . .
Measuring Spacetime

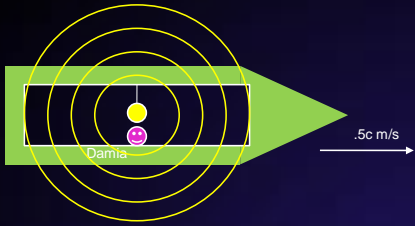


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Special Relativity & Simultaneity

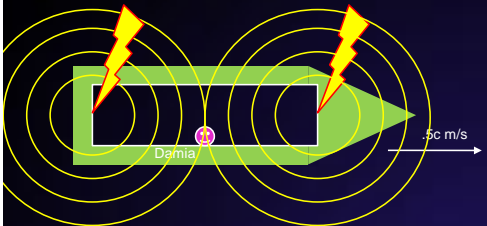
- Two events that are simultaneous in one frame of reference need not be simultaneous when viewed from within a second frame of reference moving relative to the first frame of reference.



From Damia's point of view, the light strikes both ends of the spaceship simultaneously.

From Foster's point of view, the light strikes the rear of the ship before the front; these events ARE NOT simultaneous!!

Foster
EARTH



From Foster's point of view, the lightning strikes both ends of the spaceship simultaneously.

From Damia's point of view, the lightning strikes the front of the ship before the rear; these events ARE NOT simultaneous!!

Foster
EARTH

Special Relativity and SPACETIME



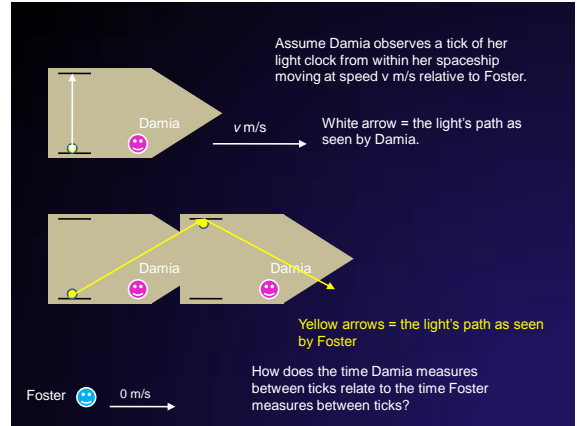
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MEASURING SPACETIME

- Two observers, side-by-side, at rest relative to one another, share the same reference frame and will also **agree on measurements** of **SPACE** and **TIME**.
- Two observers moving with respect to each other start to **disagree on their measurements** of both **SPACE** and **TIME**.
 - At "normal" speeds, the differences in measurements are imperceptibly small.
 - At speeds near the speed of light – relativistic speeds – the differences are appreciable; now each observer is in two very different parts of the continuum that is SPACETIME.

MEASURING SPACETIME

- **SPACE MEASUREMENT:** the distance between two locations whose positions are known *simultaneously*.
- What is time?
 - What clocks measure.
 - THEREFORE . . .
- **TIME MEASUREMENT:** accomplished using a clock.

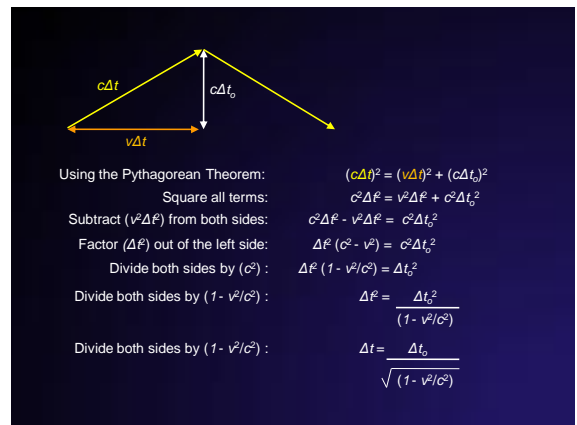
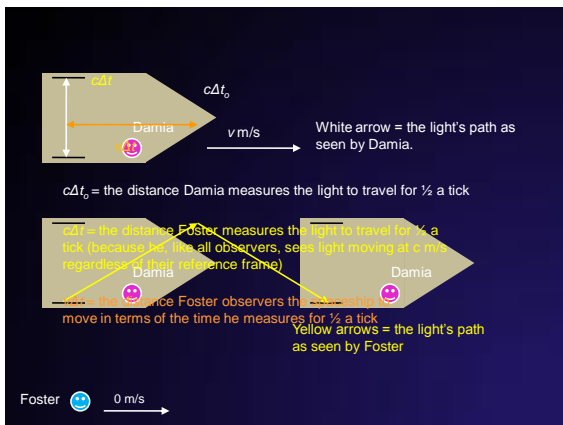


Representing the various times

- The time Damia measures = Δt_0 = PROPER TIME
- **Proper Time:**
 - the time interval between two events that occur at the same location within a single inertial reference frame.
- The time Foster measures = Δt

Using Einstein's Two Postulates

- **Principle of Relativity:**
 - The clock works the same for both observers (i.e. both Damia and Foster)
- **Constancy of the Speed of Light:**
 - A moving photon takes longer to travel a longer distance (since Foster and Damia observe different distances)



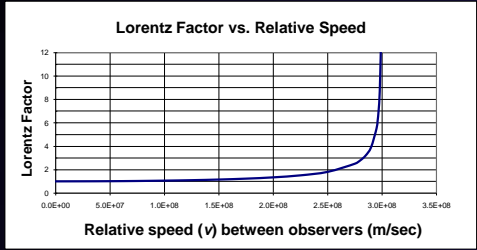
$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \text{Lorentz Factor}$$

Therefore the time Foster measures within Damia's reference frame moving relative to his =

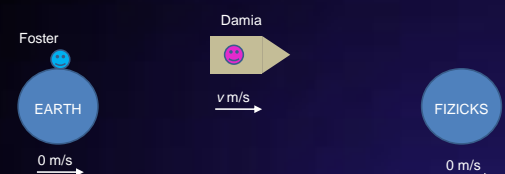
$$\Delta t = \gamma \Delta t_0$$

The Impact of γ



- γ becomes large only when v approaches c m/s.
- As a result, Δt differs from Δt_0 only when v approaches c m/s.

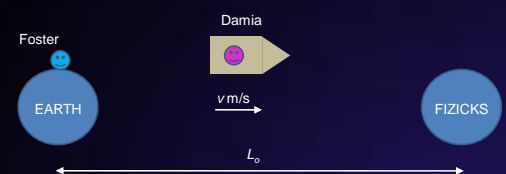
How do the measured lengths of two observers (moving relative to one another) compare?




- Observer 1 (Foster) on Earth.
- Observer 2 (Damia) on a spacecraft moving at speed v relative to the two planets.

What does each observer measure as the distance separating the two planets?

Representing the Various Distances



- Distance measured by Foster = L_0 = Proper Length
- Proper Length: the length (or distance) as measured by an observer at rest with respect to that length (or distance).
- Distance measured by Damia = L



- Consider the time required for the trip from Earth to Fizicks:
- For Foster,

$$v = (L_0)/(\Delta t)$$

$$\Delta t = (L_0)/(v)$$

- Since $\Delta t_0 = \Delta t/\gamma$, the duration of the trip that Damia measures will be shorter than what Foster measures it to be for her.
- If Damia measures less time, but both Foster and Damia agree that she moves at speed v , then Damia must also measure a smaller distance.

$$\frac{L}{\Delta t_0} = v = \frac{L_0}{\Delta t}$$

$$\frac{L \text{ (contracted length)}}{\Delta t_0 \text{ (proper time)}} = v = \frac{L_0 \text{ (proper length)}}{\Delta t \text{ (dilated time)}}$$

$$\frac{L \text{ (smaller length)}}{\Delta t_0 \text{ (smaller time)}} = v = \frac{L_0 \text{ (larger length)}}{\Delta t \text{ (larger time)}}$$

- Damia's measured distance = $L = v (\Delta t_o)$
- But $\Delta t_o = \Delta t / \gamma$
- Therefore $L = v (\Delta t / \gamma)$
- Since $\Delta t = (L_o) / (v)$
- Substituting $L = v ((L_o) / (v)) / \gamma$
- $L = (v) (L_o) / (v) (\gamma)$
- Cancelling v $L = \cancel{(v)} (L_o) / \cancel{(v)} (\gamma)$

$$L = (L_o) / (\gamma)$$

To Summarize:

- **Time Dilation:** moving clocks are MEASURED to run slowly.
- **Time Dilation:** time is observed to move slower when viewed from a second inertial reference frame. However, the moving person does not "live" any longer. In fact, to the moving person, the stationary observer's time also appears to be slower.
- **Length Contraction:** the length of a moving object (in the direction of relative motion) is MEASURED to be shorter than it is when measured at rest.
 - Dimensions perpendicular to the relative velocity are unaffected. They retain their original height or width.

To Solve Special Relativity Problems:

Variables to Identify

v
 L_o
 L
 Δt_o
 Δt

L_o = length measured by observer at rest with respect to the length

Δt_o = time measured by the observer that sees the two events at the beginning and ending times to occur at the same location

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\Delta t = (\gamma) (\Delta t_o)$$

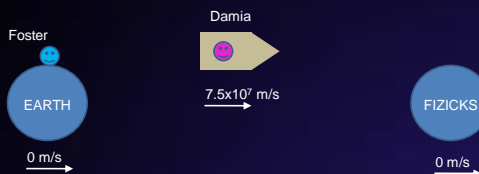
$$L = (L_o) / (\gamma)$$

$$\frac{L}{\Delta t_o} = v = \frac{L_o}{\Delta t}$$

Examples

- What will be the mean lifetime of a muon as measured in the laboratory if it is traveling 0.60c with respect to the laboratory? Its mean life at rest is 2.2×10^{-6} seconds.
- **2.8×10^{-6} seconds**
- A spaceship passes the earth at speed $v = 0.80c$. Describe the changes in length of a meter stick as it is slowly rotated from vertical to horizontal by a person inside, as viewed
 - (a) by another person in the spaceship
 - (b) by an observer on earth
 - (a) **It is one meter long in all orientations since it is at rest with respect to this second person.**
 - (b) **When vertical its length is 1 meter; when horizontal its length is 0.60m (this assumes that horizontal is the direction of the ship's motion).**

Example 3



1. Damia ages 13 years (according to her calendar) in travelling to Fizicks from Earth. In that much time, how much does she think Foster ages?
13.4 Years
2. As she arrives at Fizicks, how far does she believe she has travelled?
 $3.07 \times 10^{16} \text{m} = 3.24 \text{ Light-years (LY)}$
3. What is the actual distance between Earth and Fizicks?
 $3.18 \times 10^{16} \text{m} = 3.35 \text{ LY}$